

## 8.5 Arc Length of a Circle

SWBAT find the arc length of a circle given a central angle in radians or degrees.

**Circumference:** The circumference of a circle is the distance around the circle.

The circumference  $C$  of a circle is  $C = \pi d$  or  $C = 2\pi r$ , where  $d$  is the diameter of the circle and  $r$  is the radius of the circle.



1. The radius of a circle is 11 inches. Find the circumference of the circle and round to the nearest hundredth.

$$C = 2\pi(11) = 69.08 \text{ in}$$

2. The diameter of a circle is 4 meters. Find the circumference of the circle and round to the nearest hundredth.

$$C = 2\pi(2) = 12.56 \text{ m}$$

3. The circumference of a circle is 6 meters. Find the radius of the circle and round to the nearest hundredth.

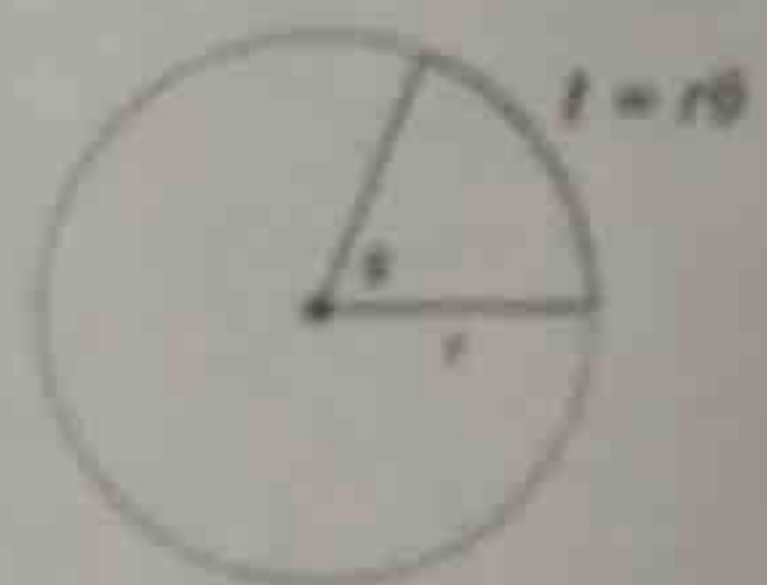
$$C = 2\pi r \quad C = 6.28 \quad 0.96 \text{ m}$$

$$r = 0.96$$

**Arc Length (in radians):** The formula for arc length in radians is  $l = r\theta$ , where  $l$  is the arc length,  $r$  is the radius, and  $\theta$  is the measure of the angle subtended by the arc, in radians.

$2\pi$  is the  $\theta$  for an entire circle.

If we want just a portion of the circumference,  $\theta$  is the measure for just that portion.



4. The radius of a circle is 1. What is the length of an arc that subtends an angle of  $\pi/3$  radians?

$$l = \frac{\pi}{3} \cdot 1 = 1.05 \text{ units}$$



5. The radius of a circle is 13 meters. What is the length of an arc that subtends an angle of  $\pi/5$  radians?

$$l = 13 \cdot \frac{\pi}{5} = \frac{13\pi}{5} = 8.164 \text{ m}$$



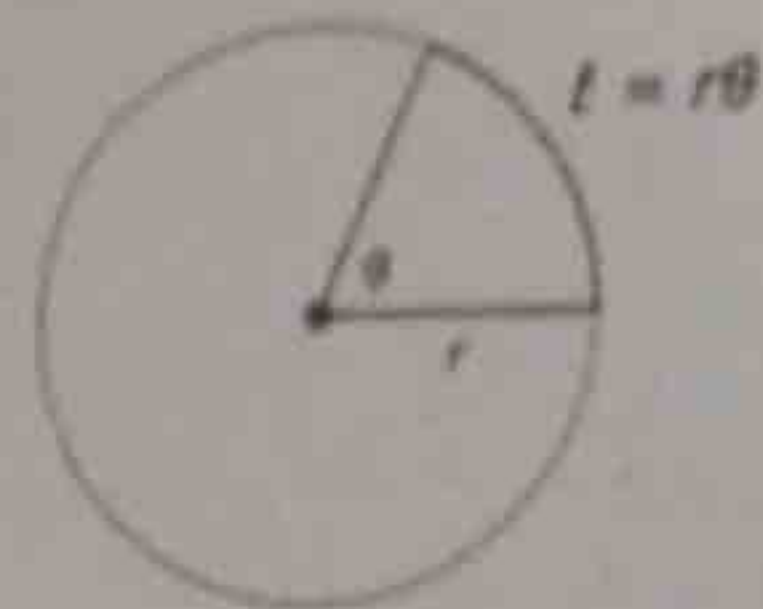
6. The radius of a circle is 17 miles. What is the length of an arc that subtends an angle of  $\pi$  radians?

$$l = 17(\pi) = 53.38 \text{ miles}$$



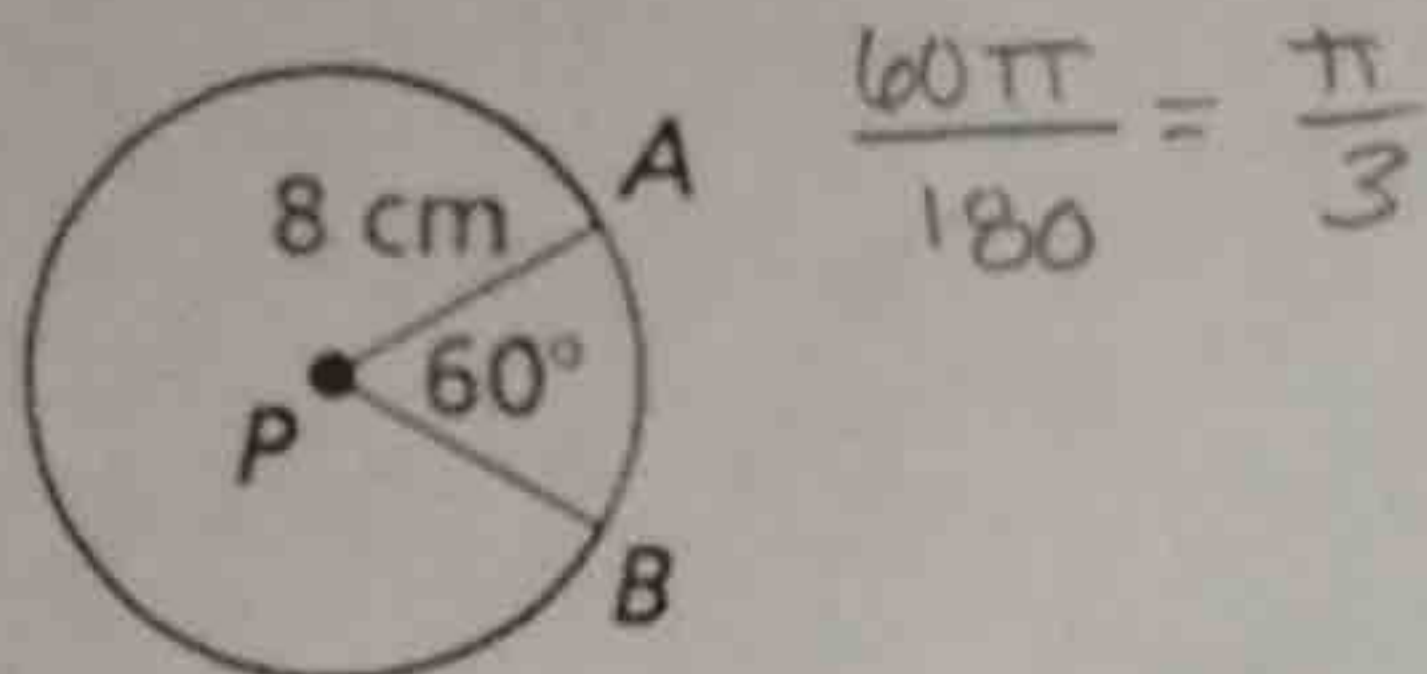
**Arc Length (in degrees):** Convert the degrees to radians, and then use the radian formula! Remember, to convert degrees to radians, multiply by:

$$\theta = \frac{\pi}{180}$$



7. Find each indicated measure. If necessary, round to the nearest tenth.

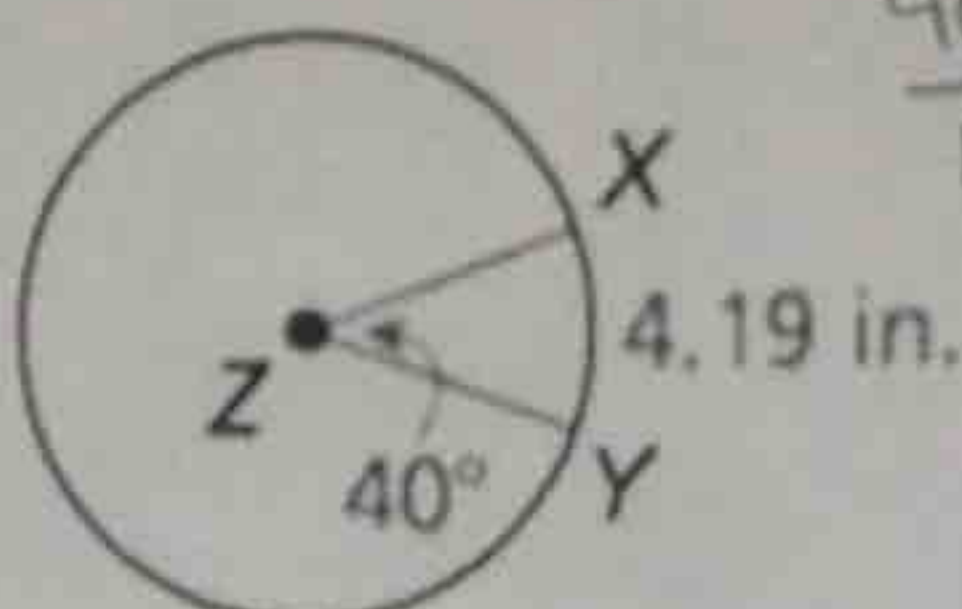
a. arc length of  $\widehat{AB}$



$$\frac{60\pi}{180} = \frac{\pi}{3}$$

$$l = 8 \left( \frac{\pi}{3} \right) = 8.37 \text{ cm}$$

b. circumference of  $\odot Z$

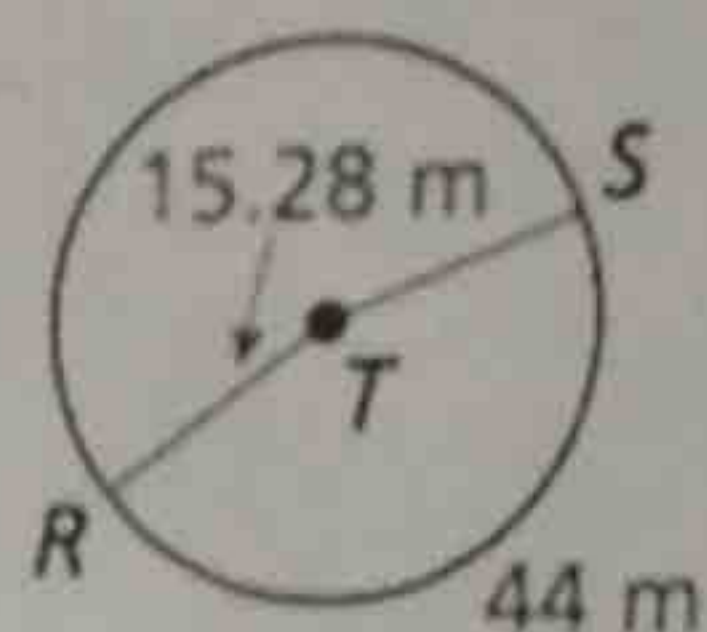


$$\frac{40\pi}{180} = \frac{2\pi}{9}$$

$$4.19 = \frac{2\pi}{9} (r)$$

$$r = 6 \quad C = 2(6)(3.14) \\ C = 37.68$$

c.  $m\widehat{RS} = 165^\circ$



$$44 = \theta(15.28)$$

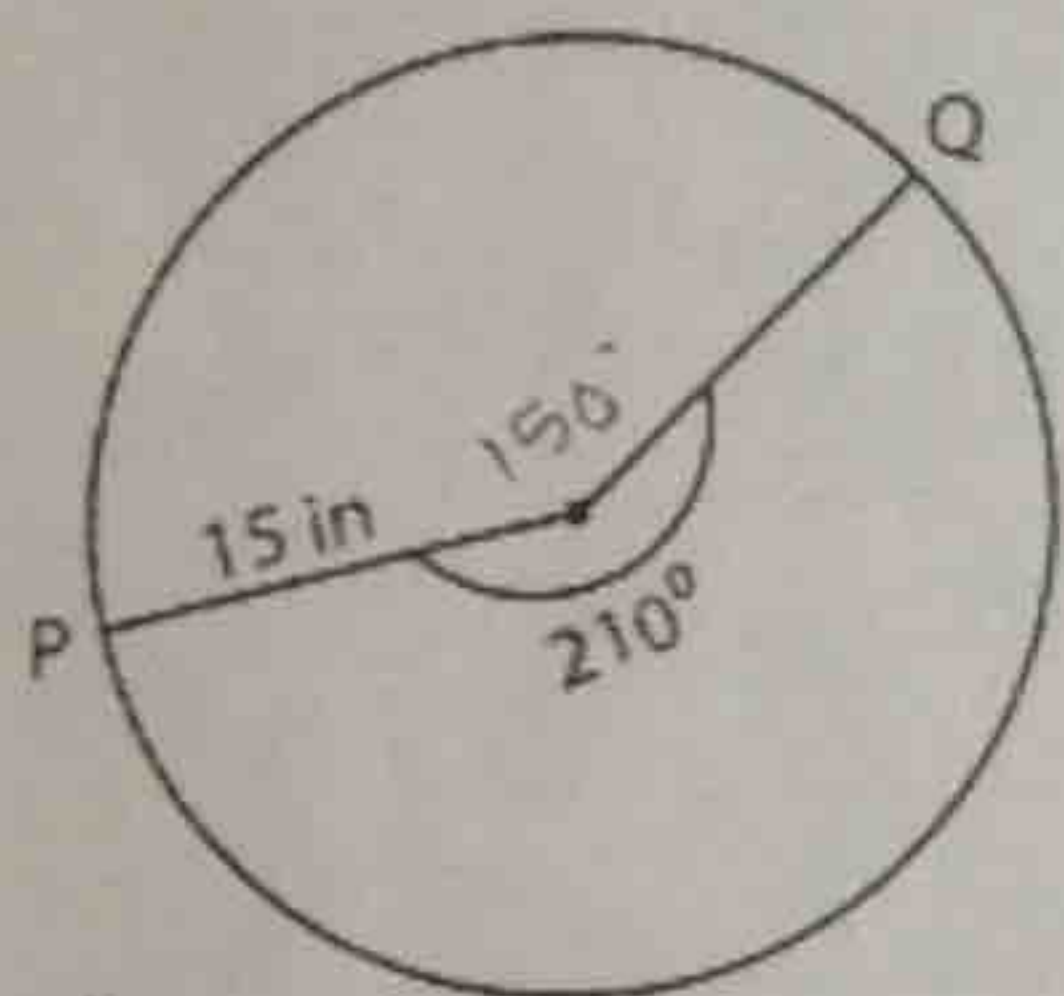
$$\theta = 2.88 \text{ rad in radians}$$

$$\frac{2.88 \cdot 180}{\pi} = 165^\circ$$

Find the missing one. Round the radius and central angle to the nearest whole number.

Round the arc length to two decimal places. (use  $\pi = 3.14$ )

1)



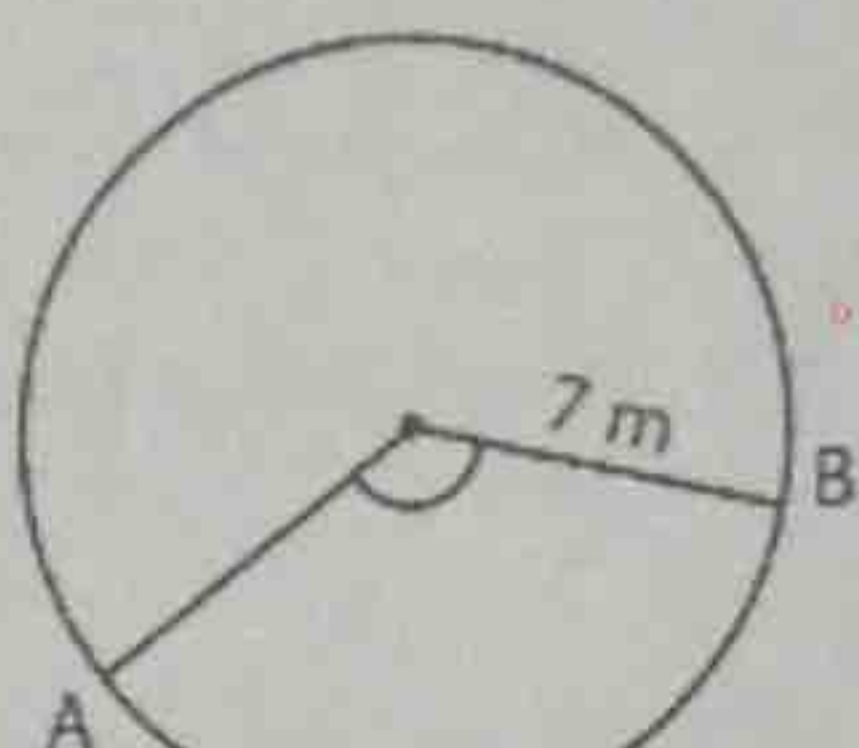
$$l = 15 \left( \frac{5\pi}{6} \right)$$

Radius = 15 in

Central angle =  $150^\circ = \frac{5\pi}{6}$

Length of the arc  $\widehat{PQ} = \underline{39.25 \text{ in}}$

2)



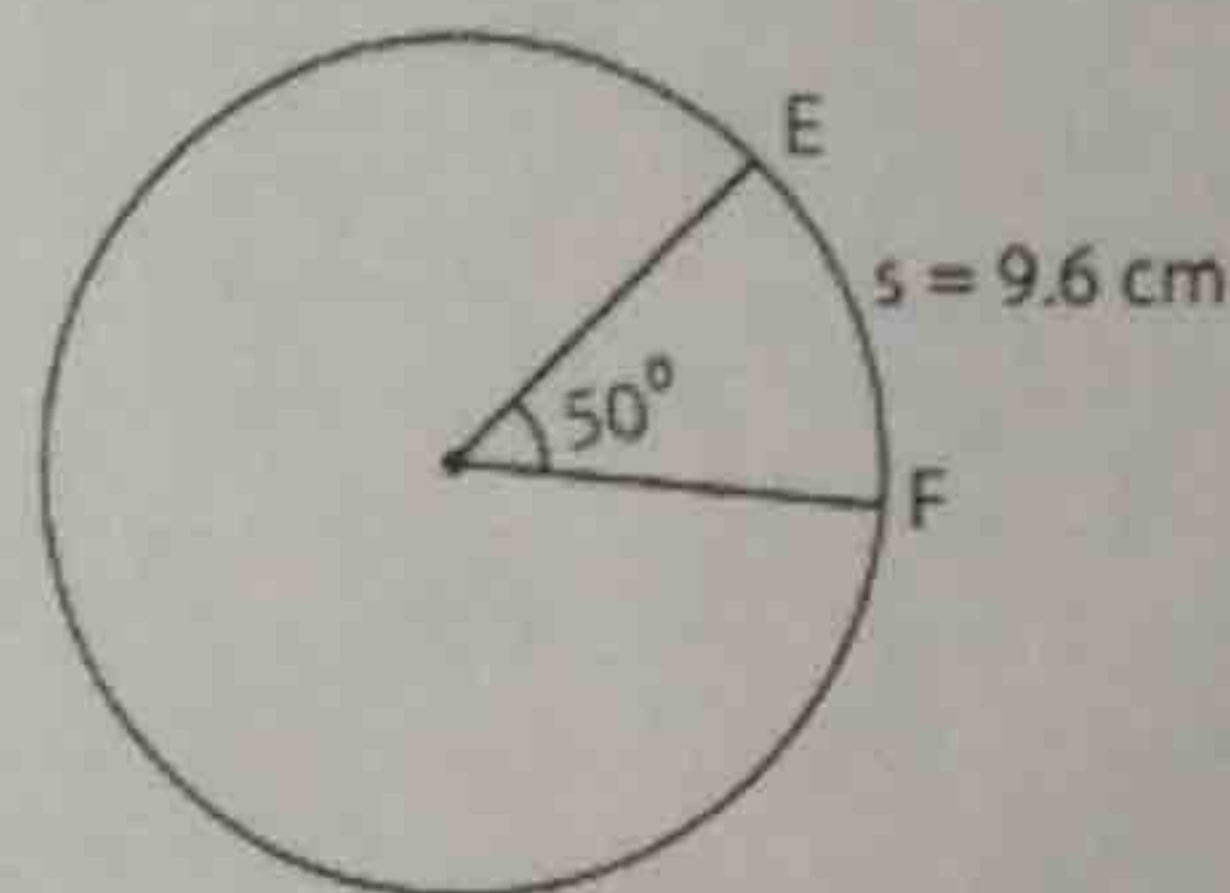
$s = 15.87 \text{ m}$

Radius = 7 m

Central angle =  $130^\circ$

Length of the arc  $\widehat{AB} = \underline{15.87 \text{ m}}$

3)



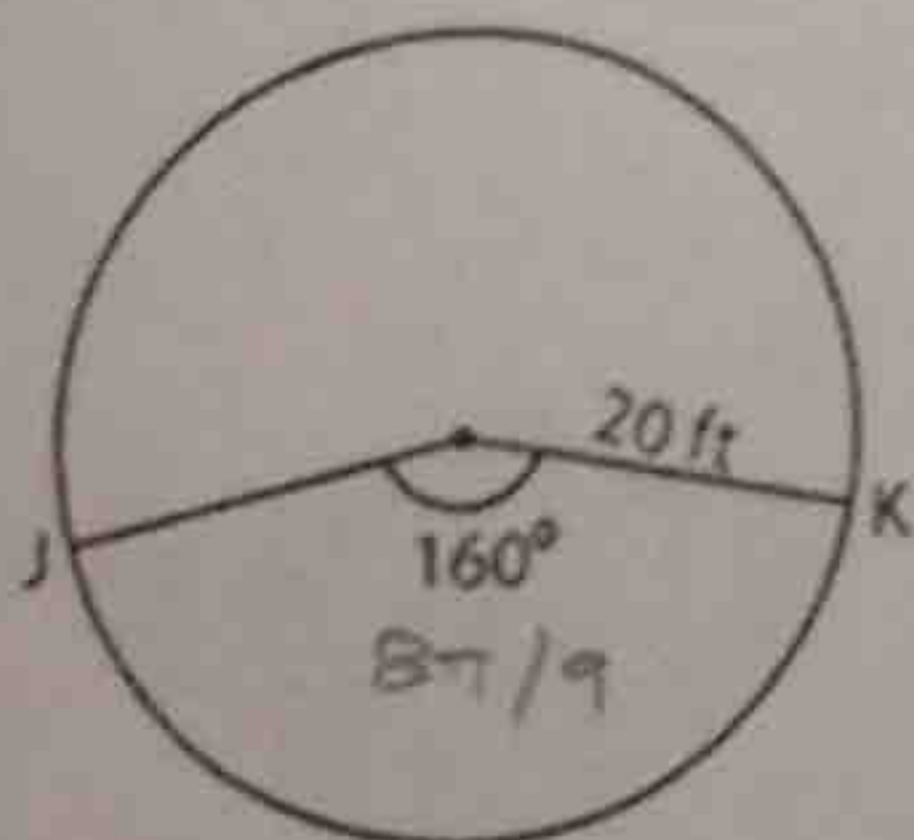
$s = 9.6 \text{ cm}$

Radius = 11 cm

Central angle =  $50^\circ = \frac{5\pi}{18}$

Length of the arc  $\widehat{EF} = \underline{9.6 \text{ cm}}$

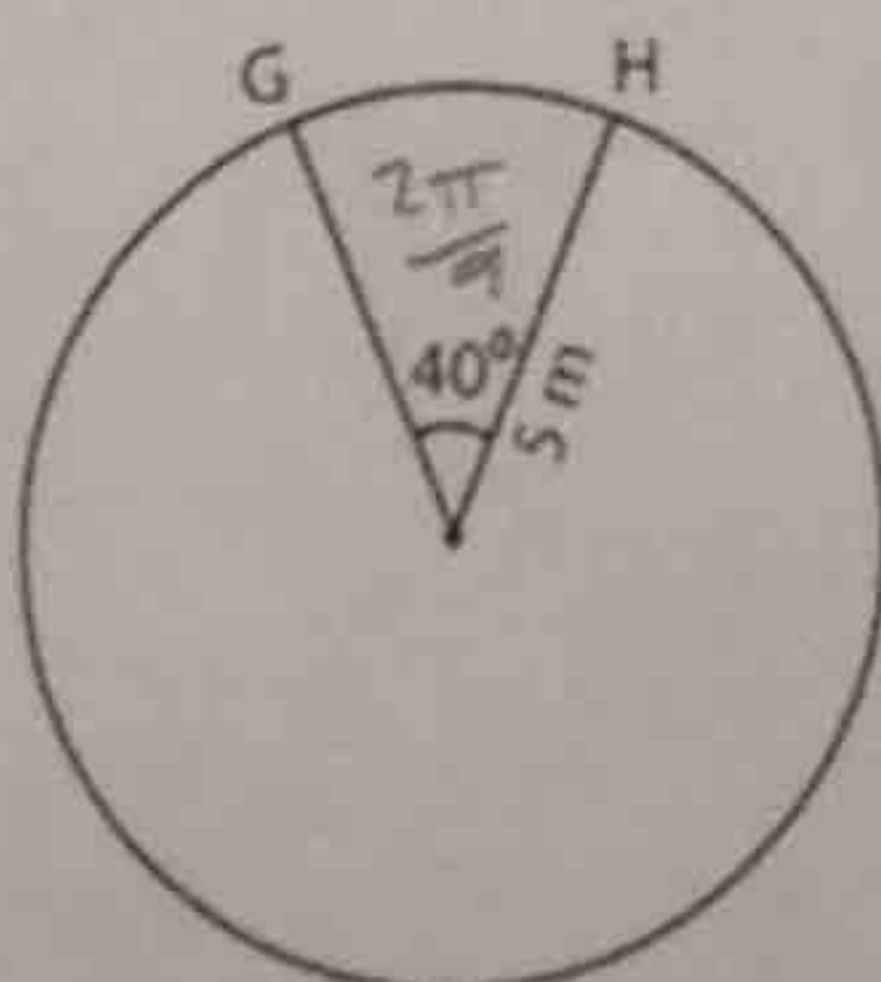
4)



Length of the arc  $\widehat{JK} = \underline{55.82}$

$$l = 20 \left( \frac{8\pi}{9} \right) = 55.82$$

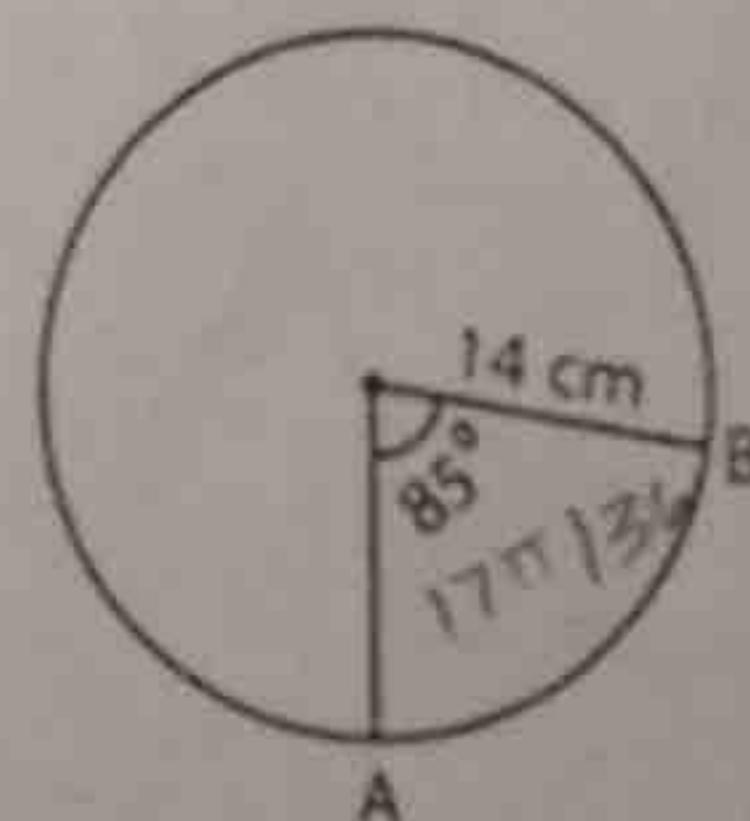
5)



Length of the arc  $\widehat{GH} = \underline{3.5 \text{ m}}$

$$l = 5 \left( \frac{2\pi}{9} \right) = 3.5$$

6)



Length of the arc  $\widehat{AB} = \underline{20.76 \text{ cm}}$

$$l = 14 \left( \frac{17\pi}{36} \right) = 20.76$$